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Using the eXtended Finite Element Method (XFEM) to Simulate Own Frequency under External Influences of a Closed System based on Dynamic Compensation Method

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Abstract

Many mechanical systems with elastic and damping properties are prone to resonant phenomena at natural frequencies under the action of external forces. Resonance phenomena can also be preserved in control systems of such objects closed by negative feedback, which reduces the quality of the functioning of these systems. The paper under review deals with the practically important problem of suppressing oscillations at natural frequencies in control systems for a linear oscillator, which is traditionally considered a dynamic object described by a second-order differential equation with complex roots of the characteristic equation. The focus of the work is the question of the roughness of the system in relation to inaccurate knowledge of the resonant frequency of the controlled object (which is typical for many technical applications).

Abstract

Banyak sistem mekanis dengan sifat elastis dan redaman yang rentan terhadap fenomena resonansi pada frekuensi alami di bawah aksi gaya eksternal. Fenomena resonansi juga dapat dipertahankan dalam sistem kontrol benda-benda yang ditutup oleh umpan balik negatif, yang mengurangi kualitas fungsi sistem ini. Makalah yang sedang ditinjau ini membahas masalah praktis yang penting untuk menekan osilasi pada frekuensi alami dalam sistem kontrol untuk osilator linier, yang secara tradisional dianggap sebagai objek dinamis yang dijelaskan oleh persamaan diferensial orde dua dengan akar-akar kompleks dari persamaan karakteristik. Fokus dari pekerjaan ini adalah pertanyaan tentang kekasaran sistem dalam kaitannya dengan pengetahuan yang tidak akurat tentang frekuensi resonansi dari objek yang dikendalikan (yang merupakan tipikal untuk banyak aplikasi teknis).

Kata kunci: Sistem mekanis, Sifat elastis dan redaman, Frekuensi alami, Matlab.

INTRODUCTION

Many mechanical systems with elastic and damping properties are prone to resonant phenomena at natural frequencies under the action of external forces. Resonance phenomena can also be preserved in control systems of such objects closed by negative feedback, which reduces the quality of the functioning of these systems. The paper under review deals with the practically important problem of suppressing oscillations at natural frequencies in control systems for a linear oscillator, which is traditionally considered a dynamic object described by a second-order differential equation with complex roots of the characteristic equation. The focus of the work is the question of the roughness of the system in relation to inaccurate knowledge of the resonant frequency of the controlled object (which is typical for many technical applications).

As an example, here we can cite the problem of controlling an object with pronounced oscillatory properties, when the parameters of the object, for one reason or another, may deviate from the calculated values. Attempts to obtain under these conditions in a closed system the aperiodic character of transient processes with the help of the corresponding desired arrangement of roots, for example, the binomial , are unlikely to be successful. In a real closed system, fluctuations will inevitably occur. In this case, it may even turn out that setting the desired location with a more stable operation of a closed system, i.e., operation without abrupt changes in processes and their quality indicators.

Formulation of the problem

As a control object, we consider a second-order oscillatory system, which is under the influence of the control and the perturbing forces F_B:

$$m\ddot{y} + b\dot{y} + cy = F + F_B \qquad (1)$$

where m is the mass; b is the coefficient of viscous friction; c - hardness.

We bring equation (1) to the standard form:

$$\ddot{y} + 2\xi\,\omega_1\dot{y} + \omega_1^2 y = u + \varphi \qquad (2)$$

where $\omega_1 = \sqrt{\frac{c}{m}}$ is the natural frequency; $\xi = \frac{b}{2\sqrt{cm}}$ – dimensionless damping coefficient ($\xi < 1$); $u = \frac{F}{m}$– control; $\varphi = \frac{F_B}{m}$- perturbation.

The block diagram of the control system for the problem of stabilizing system (2) is shown in Fig.1.

In accordance with equation (2), the calculated transfer function of the control object has the form:

$$G(p) = \frac{1}{p^2+2\xi\omega_1 p+\omega_1^2} \qquad (3)$$

We assume that the calculated value without the dimensional damping coefficient ξ is equal to the value ξ^*, while the actual value of the coefficient ξ in the object equation (3) may differ from the calculated one. It is required to build a controller with a fixed setting, having a transfer function $W(p)$ and providing for a closed system:

-Effective vibration stabilization;

-Low sensitivity of the properties of a closed system to a change in the coefficient ξ

METHOD

We require that the desired transfer function of the closed system has the form:

$$H^d(p) = \frac{\alpha_2}{p^2+\alpha_1 p+\alpha_2} \qquad (4)$$

where the coefficients of the characteristic polynomial α_1 and α_2 determine the desired location of the roots.

The transfer function (4) has the minimum order $n = 2$, which ensures the physical realizability of the controller, and satisfies the astatism condition $H^d(0) = 1$. To determine the transfer function of the controller W(p), we use the dynamic compensation method. Substituting expressions (3) and (4) into the formula: $W(p) = \frac{H^d}{G[1-H^d]}$ we get:

$$W(p) = \alpha_2 \frac{p^2+2\xi\omega_1 p+\omega_1^2}{(p+\alpha_1)p} \qquad (5)$$

To take into account the influence of the perturbation φ, we write the differential equation of the closed system:

$$Q(s)y(t) = \alpha_2(s^2 + 2\xi\omega_1 s + \omega_1^2)y^*(t) + (s+\alpha_1)s\varphi(t) \qquad (6)$$

with $Q(s) = (s^2 + \alpha_1 s + \alpha_2)(s^2 + 2\xi\omega_1 s + \omega_1^2)$ – characteristic polynomial of a closed system at calculated values of parameters.

Equation (6) shows that for the calculated values of the parameters, the properties of the closed system according to the assignment y^*(t) are determined by the transfer function (4). At the same time, the form of the transfer function of the closed system with respect to the perturbation:

$$H'(p) = \frac{(p+\alpha_1)p}{(p^2+\alpha_1 p+\alpha_2)(p^2+2\xi\omega_1 p+\omega_1^2)} \qquad (7)$$

Shows that for any limited perturbation in a closed system, oscillations occur at the natural frequency $\omega_1$of the control object (2), even if the desired characteristic polynomial $p^2+\alpha_1 p + \alpha_2$in (7) has real roots.

Note that oscillations in a closed system will also take place under nonzero initial conditions, even if there is no perturbation φ.

A similar result, characteristic of the dynamic compensation method, has already been discussed.

Let us consider the behavior of a closed system with controller (5) when the parameters of the control object deviate from the calculated values.

If the real transfer function of the object has the form.

$$\tilde{G}(p) = \frac{1}{p^2+2\tilde{\xi}\tilde{\omega}_1 p+\tilde{\omega}_1^2} \quad (8)$$

Then for a closed system instead of (4) we get:

$$\tilde{H}(p) = \frac{\alpha_2(p^2+2\xi\omega_1 p+\omega_1^2)}{\tilde{Q}(p)} \quad (9)$$

where $\tilde{Q}(p) = \left(p^2 + 2\tilde{\xi}\tilde{\omega}_1 p + \tilde{\omega}_1^2\right)(p+\alpha_1)p + \alpha_2(p^2 + 2\xi\omega_1 p + \omega_1^2)$

Analysis of the characteristic polynomial $\tilde{Q}(p)$shows that small deviations of the parameters $\Delta\xi = \tilde{\xi} - \xi$ and $\Delta\omega_1 = \tilde{\omega}_1 - \omega_1$ will not lead to abrupt changes in processes in a closed system. However, oscillations at natural frequency ω_1 are still inevitable.

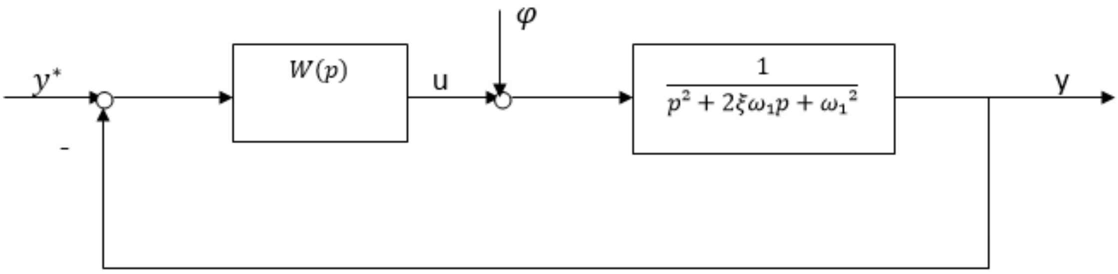

Fig.1. Block diagram of the control system

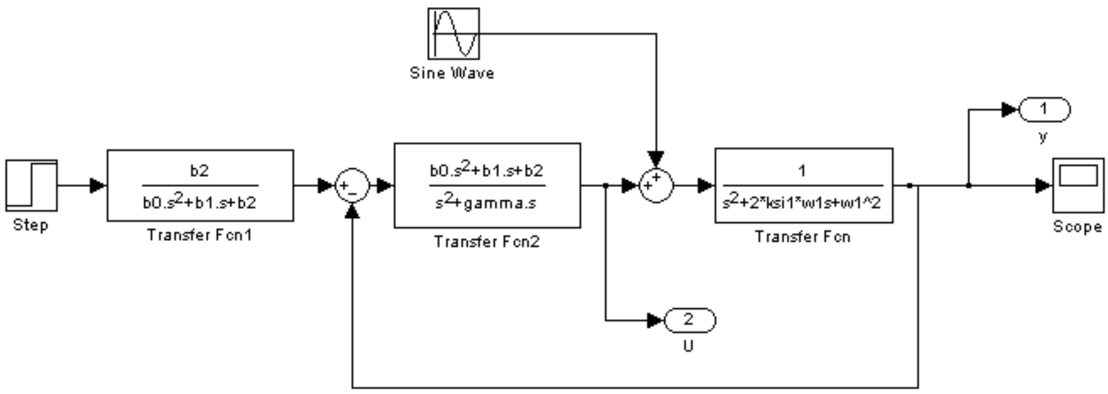

Fig.2. Structural diagram of control with prefilter

RESULTS AND DISCUSSION

As a control object, we consider a second-order oscillatory system (2). We will assume that the dimensionless damping coefficientξ, whose nominal value is equal to ξ^* (ξ^*<1), can vary within certain limits. It is required to find the transfer function $W(p)$ of the controller, which provides for the control system, the block diagram of which is shown in Fig.1, low sensitivity with respect to small deviations of the parameter $\xi$of the control object from the nominal value ξ^*.

. Let us first consider an auxiliary problem of optimal stabilization of a second-order oscillatory system with respect to the integral quadratic performance index of the type:

$$J = \int_0^\infty (x_1^T Qx + u^T Ru)dt \qquad (10)$$

This problem is necessary to justify the choice of weight matrices Q and R from (10).

Let us write the equation of the control object (2) at the nominal value of the parameter$\xi = \xi^*$ in the state variables$x_1 = y, x_2 = \dot{y}$:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\omega_1^2 x_1 - 2\xi^* \omega_1 x_2 + u + \varphi \end{cases} \qquad (11)$$

Optimality will be understood in the sense of the minimum of the integral of the weighted sum of the oscillation energy and the energy spent on control:

$$J = \int_0^\infty (\omega_1^2 x_1^2 + x_2^2 + ru^2)dt \qquad (12)$$

where the weighting factor $r > 0$ is not yet fixed.

The initial and final conditions for the stabilization problem have the form:

$$x_1(0) = x_{10}, x_2(0) = x_{20}, x_{1\infty} = 0, x_{2\infty} = 0$$

The optimization problem (11), (12) will be solved by the Lagrange method. Using the factors λ_1 and λ_2, we compose an auxiliary functional:

$$\tilde{J} = \int_0^\infty [\omega_1^2 x_1^2 + x_2^2 + ru^2 + \lambda_1(\dot{x}_1 - x_2) + \lambda_2(\dot{x}_2 + \omega_1^2 x_1 + 2\xi^* \omega_1 x_2 - u)]\, dt \qquad (13)$$

The necessary condition for the extremum of the functional (13), which consists in the vanishing of its first variation, leads to the system of Euler-Lagrange equations:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\omega_1^2 x_1 - 2\xi^* \omega_1 x_2 + u \\ \dot{\lambda}_1 = 2\omega_1^2 x_1^2 + \omega_1^2 \lambda_2 \\ \dot{\lambda}_2 = 2x_2 - \lambda_1 + 2\xi^* \omega_1 \lambda_2 \\ 2\text{ru} - \lambda_2 = 0 \end{cases} \qquad (14)$$

From the last equation of system (14), which is the condition $\frac{\partial \tilde{J}}{\partial u}$ =0 that the partial derivative of the functional $\tilde{J}$vanishes with respect to the explicitly incoming control u , we find:

$$u = \frac{\lambda_2}{2r} \qquad (15)$$

Taking into account (15), we rewrite the remaining four equations from (14) in the form:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\omega_1^2 x_1 - 2\xi^* \omega_1 x_2 + \frac{1}{2r}\lambda_2 \\ \dot{\lambda}_1 = 2\omega_1^2 x_1^2 + \omega_1^2 \lambda_2 \\ \dot{\lambda}_2 = 2x_2 - \lambda_1 + 2\xi^* \omega_1 \lambda_2 \end{cases} \qquad (16)$$

We write the matrix A of system (16):

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\omega_1^2 & -2\xi^* \omega_1 & 0 & \frac{1}{2r} \\ 2\omega_1^2 & 0 & 0 & \omega_1^2 \\ 0 & 2 & -1 & 2\xi^* \omega_1 \end{bmatrix} (17)$$

And its characteristic equation:

$$\det[\ \nu E - A\ ] = \det \begin{bmatrix} \nu & -1 & 0 & 0 \\ \omega_1^2 & \nu + 2\xi^* \omega_1 & 0 & \frac{-1}{2r} \\ -2\omega_1^2 & 0 & \nu & -\omega_1^2 \\ 0 & -2 & 1 & \nu - 2\xi^* \omega_1 \end{bmatrix} = \nu^4 + 2B\nu^2 + C = 0$$

(18)

where $B = \omega_1^2(1 - 2\xi^*) - \frac{1}{2r}$, $C = \frac{\omega_1^2}{r} + \omega_1^4 > 0$ -coefficients depending on the parameters $\xi^*$and ω_1 of the control object and the value of the weight factor r from the functional (4).

Let us return to the block diagram of the control system shown in Fig. 2. Let us find the transfer function $\boldsymbol{W}(\boldsymbol{p})$ of a physically realizable controller of the minimum order, which provides for the closed system astatism of the first order and the desired characteristic polynomial (18). The solution is determined by the same formulas (4), (5) as in section 3, in which the parameters $\boldsymbol{\alpha_i}$take the values $\boldsymbol{\alpha_i^*}(\boldsymbol{i} = \overline{\mathbf{1,4}})$.

Recall that in section 3 the controller (4) was tuned in accordance with the standard (binomial) form of the fourth order (6), and in the considered example, in accordance with the polynomial (14) calculated by the criterion (15) minimizing not only the main motion (for $\xi = \xi^*$), but also for additional motion (for $\xi \neq \xi^*$).

Note that in both cases it is expedient to introduce a prefilter with the transfer function into the control system:

$$L(p) = \frac{\beta_2}{\beta_0 p^2 + \beta_1 p + \beta_2} \qquad (19)$$

Excluding the influence of the numerator of the transfer function of the controller (5) on the indicators of the quality of transient processes. The corresponding block diagram is shown in Fig. 2. In this case, the differential equation of the closed system will have the form:

$$Q(s)y(t) = \alpha_4 y^* + (s+\gamma)s\varphi(t) \qquad (20)$$

where $Q(s) = s^4 + \alpha_1 s^3 + \alpha_2 s^2 + \alpha_3 s + \alpha_4$-characteristic polynomial of a closed system.

Thus, for the control system, the block diagram of which is shown in Fig. 2, three options for setting up a regulator with a gear ratio (5) are considered. In all cases, the differential equation of a closed system has the same form (20). For different options for adjusting the controller, the following characteristic polynomials are obtained:

-Adjustment by compensation method:

$$Q(s) = (s^2 + 2\xi^*\omega_1 s + \omega_1^2)(s^2 + \alpha_1 s + \alpha_2) \qquad (21)$$

Recall the parameters α_i^* remained free and it was recommended to use standard forms (binomial, Butterworth, etc.) to determine them.

Let us reduce the polynomials (21) to a common geometric mean root $\omega_0 = \sqrt{\omega_1\sqrt{\gamma^2 + \delta^2}}$ and find their roots with the above numerical data. In this case, we choose the binomial form as the standard one for the first two cases. The options for the location of the roots of the characteristic polynomial of a closed system are shown:

$$\lambda_{1,2} = (-3 \pm j29{,}85)\ 1/s\ ,\ \lambda_{3,4} = -35{,}68\ 1/s \qquad (22)$$

Let us present the main results of computer simulation below:

With $\xi^* = 0{,}15$ (Fig.3)

Transfer function:

```
          1.146e006
----------------------------------------------------
s^4 + 80.35 s^3 + 2182 s^2 + 7.185e004 s + 1.146e006
```

Roots:

```
 -35.9298 + 8.6759i
 -35.9298 - 8.6759i
  -4.2464 +28.6430i
  -4.2464 -28.6430i
```

With$\xi^* = 0{,}1$ (Fig.4)

Transfer function:

```
          1.146e006
----------------------------------------------------
s^4 + 77.35 s^3 + 2179 s^2 + 7.185e004 s + 1.146e006
```

Roots:

-3.0000 +29.8496i
-3.0000 -29.8496i
-35.6762
-35.6762

With $\xi^* = 0{,}05$ (Fig.5)

Transfer function:

1.146e006

----

s^4 + 74.35 s^3 + 2176 s^2 + 7.185e004 s + 1.146e006

Roots:
-1.6357 +30.8712i
-1.6357 -30.8712i
-43.5732
-27.5077

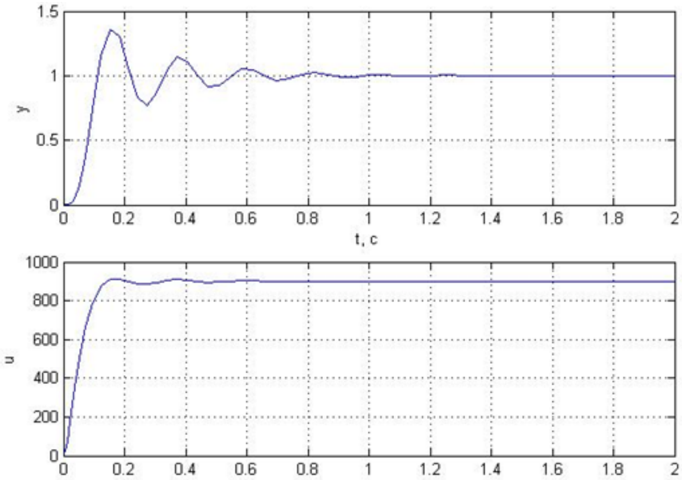

Fig.3. Results of computer simulation ($\xi^* = 0, 15$)

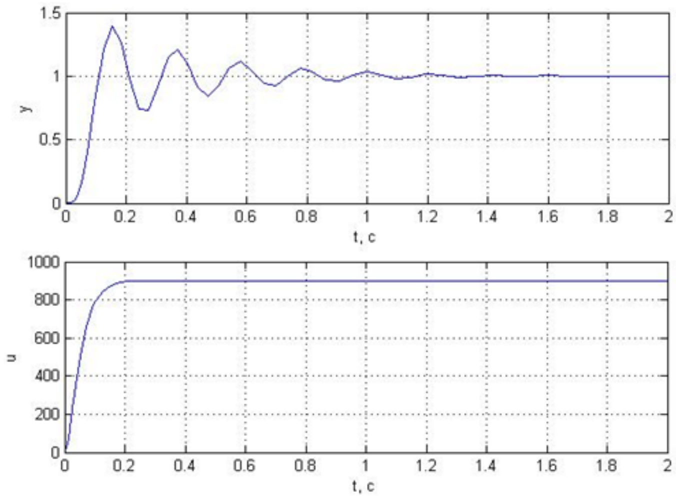

Fig.4. Results of computer simulation ($\xi^* = 0, 1$)

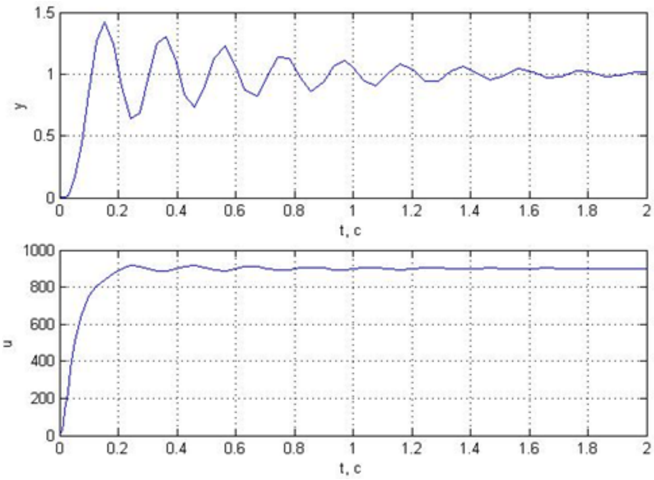

Fig.5. Results of computer simulation ($\xi^* = 0,05$)

A comparative analysis of the variants shows that the first variant has the high parametric sensitivity, for which the characteristic polynomial (21) does not take into account the properties of the control object.

CONCLUSION

In this study, the controller calibration method has been analyzed for the problem of linear oscillator stability. It has been shown that the sensitivity function method has a low sensitivity to changes in the parameters of the linear oscillator. The implementation of the method is performed using the MATLAB software system on the canonical quadratic-linear optimization problem.

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