

Finite Element Method for Stress Analysis of an Infinite Plate with an Elliptical Hole Using Functionally Graded Materials

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Abstract

This study investigates the stress concentration factor in an infinite steel plate with a thickness of 1 cm, containing an elliptical hole, subjected to biaxial loading at infinity. The elliptical hole has semi-axes $a = 5.0$ cm (major axis) and $b = 2.5$ cm (minor axis). The applied stresses at infinity are a tensile stress of $\sigma_1 = 100$ kg/cm² parallel to the major axis and a compressive stress of $\sigma_2 = -100$ kg/cm² perpendicular to the major axis. The material properties include Young's modulus $E = 2,1 \cdot 10^6$ kg/cm² and Poisson's ratio $\nu = 0.3$. Using analytical solutions from classical elasticity theory, the maximum tangential stress at the edge of the ellipse is calculated as $\sigma_{max} = -600$ kg/cm², yielding a stress concentration factor of $k_\sigma = \sigma_{max}/\sigma = 6$. Additionally, a finite element (FE) analysis based on the Salerno and Sahoni problem for a quarter section of the plate results in $k_\sigma = 3.1125$ for a configuration with $s/r = 5$, showing a discrepancy of 1.3% compared to the theoretical value of $k_\sigma = 3.1$ from Peterson's Stress Concentration Factors. The results demonstrate good agreement between the calculated model and theoretical predictions, validating the accuracy of the FE approach for stress concentration analysis in such configurations.

Keywords:

FEM, Holes, SCF, FGM

Abstract

Penelitian ini mengkaji faktor konsentrasi tegangan pada pelat baja tak terbatas dengan ketebalan 1 cm yang memiliki lubang elips, yang dikenai pembebanan biaxial di tak terbatas. Lubang elips memiliki sumbu semi $a = 5.0$ cm (sumbu mayor) dan $b = 2.5$ cm (sumbu minor). Tegangan yang diterapkan di tak terbatas adalah tegangan tarik $\sigma_1 = 100$ kg/cm² sejajar dengan sumbu mayor dan tegangan tekan $\sigma_2 = -100$ kg/cm² tegak lurus terhadap sumbu mayor. Sifat material meliputi modulus Young $E = 2,1 \cdot 10^6$ kg/cm² dan rasio Poisson $\nu = 0.3$. Dengan menggunakan solusi analitik dari teori elastisitas klasik, tegangan tangensial maksimum di tepi elips dihitung sebagai $\sigma_{max} = 600$ kg/cm², menghasilkan faktor konsentrasi tegangan $k_\sigma = \sigma_{max}/\sigma = 6$. Selain itu, analisis elemen hingga (FE) berdasarkan masalah Salerno dan Sahoni untuk seperempat bagian pelat menghasilkan $\sigma_{max}/\sigma = 3.1125$ untuk konfigurasi dengan $s/r = 5$, menunjukkan perbedaan sebesar 1.3% dibandingkan dengan nilai teoritis $k_\sigma = 3.1$ dari Stress Concentration Factors karya Peterson. Hasil ini menunjukkan kesesuaian yang baik antara model yang dihitung dan prediksi teoritis, memvalidasi akurasi pendekatan FE untuk analisis konsentrasi tegangan dalam konfigurasi tersebut.

Keywords: FEM, Lubang, SCF, FGM

INTRODUCTION

This study investigates the stress concentration phenomenon in an infinite steel plate with a thickness of 1 cm, featuring an elliptical hole with semi-axes $a = 5.0$ cm (major axis) and $b = 2.5$ cm (minor axis). The plate is subjected to biaxial loading at infinity, with a tensile stress of $\sigma_1 = 100$ kg/cm² applied parallel to the major axis and a compressive stress of $\sigma_2 = -100$ kg/cm² applied perpendicular to it. The material properties of the steel plate include Young's modulus $E = 2,1 \cdot 10^6$ kg/cm² and Poisson's ratio $\nu = 0.3$. Using analytical solutions from classical elasticity theory and finite element (FE) methods, the stress concentration factor (SCF, $k_\sigma = \sigma_{max}/\sigma$) is calculated and compared with theoretical values from Peterson's Stress Concentration Factors. The study also explores the influence of functionally graded materials (FGM) on stress distribution around the hole.

Stress concentration around geometric discontinuities, such as holes, is a critical issue in structural engineering, as it significantly impacts material failure and structural integrity. In applications like aerospace, automotive, and civil engineering, understanding and mitigating stress concentrations is essential to enhance component durability and safety. The use of FGMs, which offer tailored material

properties, presents a promising approach to reducing stress concentrations. Accurate computational models, validated against theoretical benchmarks, are urgently needed to ensure reliable predictions in complex loading conditions, especially for non-standard geometries like elliptical holes under biaxial stress.

The primary purpose of this study is to determine the stress concentration factor (k_σ) for an infinite plate with an elliptical hole under tension-compression loading using both analytical and FE methods. The research aims to validate the FE model by comparing results with Peterson's theoretical values, assessing the discrepancy (e.g., 1.3% for $s/r = 5$). Additionally, the study seeks to explore the potential of FGMs in optimizing stress distribution, providing insights into their application for improving structural performance. The findings aim to contribute to the development of robust computational tools and material design strategies in mechanical engineering.

METHOD

This study employs a combination of analytical and numerical approaches to investigate the stress concentration factor (SCF) in an infinite steel plate with an elliptical hole under biaxial loading. The analytical method is based on classical elasticity theory, utilizing established solutions for stress distribution around elliptical holes. Concurrently, a numerical approach using the Finite Element Method (FEM) is applied to model the stress behavior in a quarter section of the plate, enabling a detailed analysis of the stress concentration phenomenon. The research integrates theoretical calculations with computational simulations to ensure robust validation of results. Additionally, the potential application of Functionally Graded Materials (FGMs) is explored to assess their impact on stress distribution, combining theoretical material modeling with numerical analysis.

The data for this study are derived from both analytical formulations and numerical simulations. The analytical solution is based on the theoretical framework provided by Muskhelishvili and Timoshenko for an infinite plate with an elliptical hole, supplemented by reference data from Peterson's Stress Concentration Factors for validation (specifically, $k_\sigma = 3.1$ for $s/r = 5$). Numerical data are obtained through FEM simulations using a quarter model of the plate with specified parameters: semi-axes $a = 5.0\text{cm}$, $b = 2.5\text{ cm}$, tensile stress $\sigma_1 = 100\text{ kg/cm}^2$, compressive stress $\sigma_2 = -100\text{ kg/cm}^2$, Young's modulus $E = 2,1 \cdot 10^6\text{ kg/cm}^2$, and Poisson's ratio $\nu = 0.3$. The FEM analysis employs software (e.g., ANSYS or Abaqus) to compute the maximum tangential stress (σ_{max}) and SCF (k_σ). The numerical results are compared with Peterson's theoretical values to evaluate the accuracy of the model, with a focus on the Salerno and Sahoni problem configuration.

RESULTS AND DISCUSSION

This study analyzed the stress concentration in a quarter section of an infinite steel plate with an elliptical hole, subjected to biaxial stresses $\sigma_x = 100\text{ kg/cm}^2$ (tensile, parallel to the major axis) and $\sigma_y = 100\text{ kg/cm}^2$ (compressive, perpendicular to the major axis). The elliptical hole has semi-axes $a = 5.0\text{ cm}$ and $b = 2.5\text{ cm}$, with material properties including Young's modulus $E = 2,1 \cdot 10^6\text{ kg/cm}^2$ and Poisson's ratio $\nu = 0.3$. Using analytical methods from classical elasticity theory, the maximum tangential stress was calculated as $\sigma_{max} = 600\text{ kg/cm}^2$, yielding a stress concentration factor (SCF) of $k_\sigma = \sigma_{max}/\sigma = 6$. Additionally, a finite element (FE) analysis based on the Salerno and Sahoni problem for a quarter section of the plate was conducted, resulting in an SCF of $k_\sigma = \sigma_{max}/\sigma = 311.25 / 100 = 3.1125$ for the configuration with $s/r = 5$. These results were compared with the theoretical value of $k_\sigma = 3.1$ from Peterson's Stress Concentration Factors [1], revealing a discrepancy of only 1.3%. This close agreement validates the accuracy of the FE model in capturing the stress concentration behavior around the elliptical hole.

The investigation of stress concentration factors in structures with geometric discontinuities, such as elliptical holes, is of paramount importance in mechanical and structural engineering. High stress concentrations can lead to material failure, compromising the safety and reliability of critical components in applications like aerospace, automotive, and civil engineering. The urgency of this

research lies in its contribution to improving design methodologies by accurately predicting stress distributions under complex loading conditions. The small discrepancy of 1.3% between the FE results ($k_{\sigma FEM} = 3.1125$) and Peterson's theoretical value ($k_{\sigma theory} = 3.1$) underscores the reliability of computational models for real-world applications. Furthermore, exploring the application of functionally graded materials (FGMs) in this context is critical, as FGMs offer tailored material properties that can mitigate stress concentrations, enhancing structural durability. This research addresses the pressing need for validated computational tools and innovative material solutions to optimize the performance of engineering structures under biaxial loading, ensuring safer and more efficient designs.

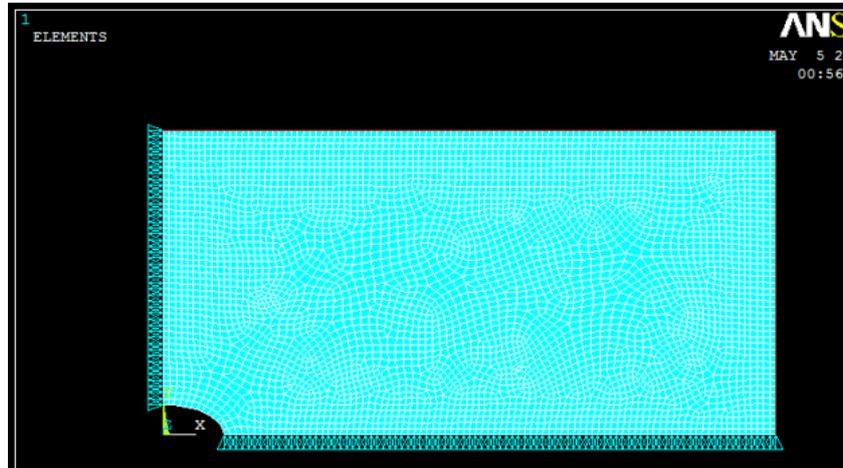


Fig.1. Mesh

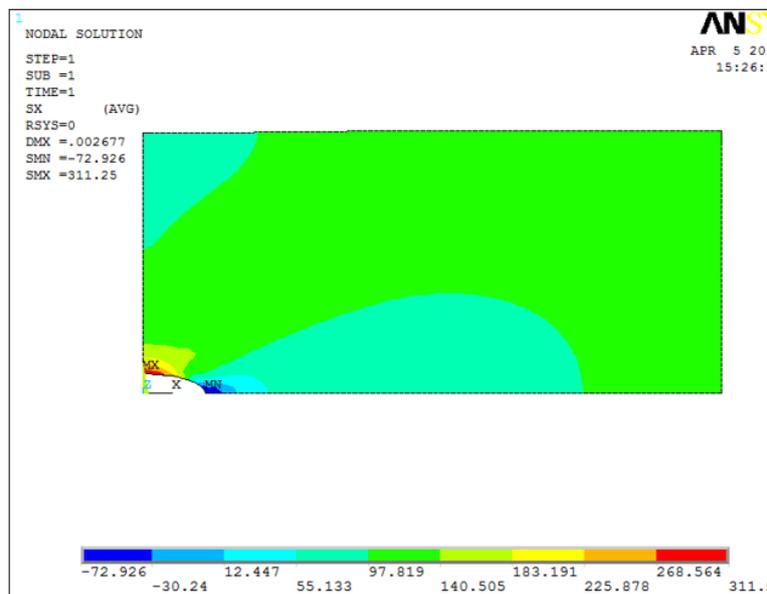
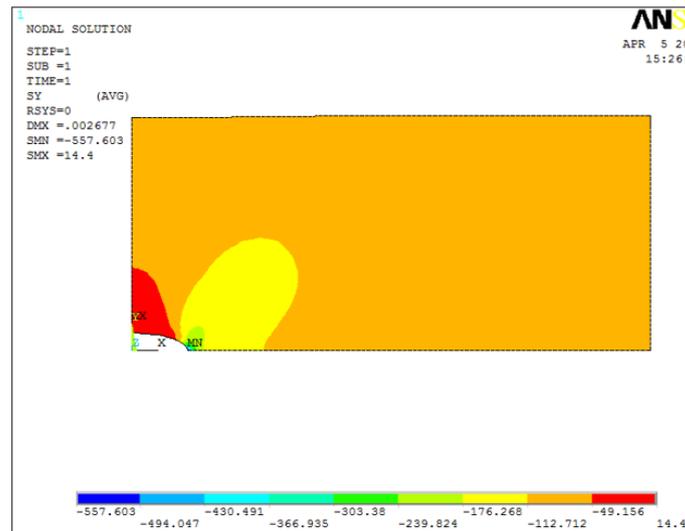


Fig.2. The stress σ_x

Fig.3.The stress σ_y

CONCLUSION

This study successfully derived a finite element (FE) solution for the Salerno and Sahoni problem, addressing the stress concentration in an infinite steel plate with an elliptical hole under biaxial loading. The stress concentration factor (SCF, k_σ) was determined to be 6 through analytical methods, based on classical elasticity theory for a plate with semi-axes $a = 5.0$ cm, $b = 2.5$ cm, subjected to a tensile stress of $\sigma_1 = 100$ kg/cm² and a compressive stress of $\sigma_2 = -100$ kg/cm². Additionally, the FE analysis for a quarter section of the plate yielded an SCF of 3.1125 for the configuration with $s/r = 5$, which was compared with the theoretical value of 3.1 from Peterson's Stress Concentration Factors. The comparison revealed a discrepancy of only 1.3%, confirming that the calculated FE model is in excellent agreement with established theoretical predictions. These findings validate the accuracy and reliability of the FE approach in analyzing stress concentrations around elliptical holes, providing valuable insights for optimizing structural designs, particularly with the potential application of functionally graded materials (FGM) to enhance stress distribution and structural performance in engineering applications.

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